DIFFERENTIAL EQUATIONS

Differential eqⁿ :- An equation of the form $f(x, y, y', y'', \ldots, y^n) = 0$ which contains an independent variable 'x', a dependent variable 'y' and derivatives of dependent variable of any order is called Differential eqⁿ ——— (1)

OR

A differential eqⁿ is an equation which involves differential coefficients or differentials.

In other words, An equation involving derivative of dependent variable with respect to independent variable is called a differential eqⁿ.

(i) $\frac{dy}{dx} - 2x - 2 = 0$

(ii) $y'' + 3y' + 2 = 0$

(iii) $y' - \sin x - e^x = 0$

(iv) $y'' - 3(y')^3 + 7y - \cos x = 0$

(v) $y = xy' + \frac{1}{y'}$

Order of Differential eqⁿ :- The order of a differential eqⁿ is the order of the highest order derivative occurring in the eqⁿ.

e.g. :- The order of (i), (ii), (iii), (iv), (v) respectively

1, 2, 1, 2, 1

Scanned with CamScanner

Degree of a Differential Equation:-

The degree of a Differential eqⁿ is the degree of the highest order differential coefficient after making the differential eqⁿ free from radicals and fractions in its derivatives and putting it in the form of the polynomial equation in its derivatives

e.g:- The degree of (i), (ii), (iii), (iv), (v) respectively

1, 1, 1, 1, 2

Linear and non-Linear Differential eqⁿ:-

A differential equation is said to be linear if the dependent variable and its derivatives occur only in the first degree and are not multiplied together Otherwise, it is called a Non-Linear Differential eqⁿ. (i), (iii) are Linear while others are non-Linear.

Some Exercise On above topic

Write the order, Degree, linear, non-linear of differential eqⁿ.

① $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + 2y = 0$ ——— ①

Scanned with CamScanner

Sol^n:- The given differential equation is

$$\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + 2y = 0$$

Order – 2 , Degree – 2

Non-Linear differential eq^n

Apply only concept
Def^n of order / Degree

(2) $y'' + (y')^2 + 2y = 0$ — (1)

The highest order derivative present in the given differential eq^n is y'' ∴ order is 2

Also degree of given eq^n is '1'

It's non-linear.

For degree
$(y'')^{\text{power}}$
Highest order

(3) $(y''')^2 + (y'')^3 + (y')^4 + y^5 = 0$

The highest order derivative present in the given eq^n is y''' so that order is '3'

Also degree of highest order i.e $(y''')^2$

∴ Degree of differential eq^n = 2

And given eq^n is Non-Linear.

Scanned with CamScanner

(3) $y = xy' + a\left[1 + (y')^2\right]^{3/2}$ ⇐ (4)

Firstly remove it

~~Given eqn is in radi~~.

Squaring both sides, we get

$$(y - xy')^2 = a^2\left[1 + (y')^2\right]^3$$

Now, we have the highest order derivative present in the differential eqn is y' is '1'

Also the degree of highest order derivative is $\left[(y')^2\right]^3$ ie $(y')^6$

∴ Degree - 6

Given differential eqn is Non-linear.

(4) $y = xy' + \frac{a}{y'}$

This can be written as

$$\underbrace{yy'} = x(y')^2 + a \quad \text{——} \quad (1)$$

dependent variable & their derivative multiplied together ie It's non-linear.

The highest order degivatives y'.

∴ order - 1 , Degree $(y')^2 = 2$

Scanned with CamScanner

(5) $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2} = 2$ ← Remove it firstly (5)

squaring both sides. we get

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = 2^2 = 4$$

$$\Rightarrow \left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = 4.$$

∴ Order is $(y') = 1$

Degree is $\left[(y')^2\right]^3 = (y')^6 = \underline{6}$

Given eq^n is Non-linear.

(6) $y''' + y^2 + e^{y'} = 0$

The highest order derivative is y'''

i.e order is '3'

Degree is not defined as $e^{y'}$ ← Infinite Terms

Non-Linear.

(7) $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x^2 \log\left(\frac{d^2y}{dx^2}\right)$

order is $\frac{d^2y}{dx^2}$ i.e '2'

Degree Non-defined As $\log(y'')$

Scanned with CamScanner